Closing Mon: 10.2
Closing Wed: 3.5(part 1)
Closing Fri: 3.5(part 2)

Entry Task:

In both cases, find dy/dx.

3.5 Implicit Differentiation

Motivation: Consider the unit circle

$$x^2 + y^2 = 1$$

This equation does NOT define a function. We say it *implicitly* defines more than one function.

In particular, it defines:

$$y = \sqrt{1 - x^2} \ \text{ or } \ y = -\sqrt{1 - x^2}$$

General Notes

We say $F(x,y) = 0$ implicitly defines one or more functions $y = y(x)$.

If we can think of y as a function of x, then we can differentiate $F(x,y(x)) = 0$, directly.

But, we must appropriately use the chain rule.

Inverse Functions:

We write inverse functions as

$y = f^{-1}(x)$ which is equivalent to

$f(y) = x$.

We can implicitly differentiate

$$\frac{d}{dx}[f(y) = x] \Rightarrow f'(y)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$$